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Analysis of optimum power extraction in a MHD generator with spatially varying electrical conductivity

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Abstract

Analytical expressions for optimum power extracted in an MHD generator, with constant channel width, uniform magnetic field and constant electrical conductivity have been proposed by Neuringer. In this paper, a numerical approach to study optimum power extraction in a MHD generator for an arbitrary geometry, magnetic field and electrically conductivity has been developed. This is accomplished by coupling a flow solver to an optimization code. Using this approach, Neuringer's analysis has been extended to study MHD channels with spatially varying electrical conductivity. An Euler flow solver coupled to an optimization code is used to predict fluid-dynamical variables, optimum power extracted and the voltage drop in the external load corresponding to optimum power extraction. Neuringer's results showed that in an MHD uniform channel with constant conductivity, most of the power delivered to the external load takes place near the extremities. Using our coupled flow/optimizer approach, we show that a more uniform axial power density distribution can be obtained with a spatially varying electrical conductivity distribution, thus making it a useful tool in design/analysis of practical MHD generators.

Keywords: Compressible flow; Optimization; MHD generators; Variable electrical conductivity

1. Introduction

Interest in designing and optimizing MHD generators in the sixties came from the possibility of converting the power generated from controlled nuclear fusion into electrical energy. These devices are characterized by high efficiencies and can operate at high temperatures. Several analytical studies [1–12] have investigated various aspects of design, analyses and optimization of MHD generators. Ref. [1] through [12] represents only a partial list of the large body of literature pertaining to the study of MHD generators.

While the interest in MHD-based systems subsided in the seventies and eighties, there has been renewed interest in the possible use of MHD for hypersonic flight. The AJAX concept, proposed in Russia in the mid-nineties, requires the use of a MHD generator, placed between the inlet and combustor (burner) of a hypersonic air-breathing engine [13,14]. The MHD generator would extract electrical energy from the flow kinetic energy of an ionized gas stream at the inlet and thus bypass a portion of the intake kinetic energy around the burner. This scheme allows an active control of the temperature rise associated with the deceleration of the inlet air flow while being more effective in decelerating the flow as compared to simple adiabatic compression processes. The electrical power generated could be used for powering electrical systems on-board and also for accelerating the flow stream exiting the combustor. The MHD by-pass engine thus seems like a strong candidate for future hypersonic flight applications. Hence, there is a need to explore various issues with regards to design and optimization of the MHD generator, which is the main component of the MHD by-pass system.

Several constraints/requirements need to be considered for the design and optimization of the entire MHD by-pass system. Optimization studies of the entire MHD-based power generation system, with its various sub-systems and components would be a daunting task. Analytical and/or theoretical estimates can play an important role in fixing the limits of operation of various sub-systems in a MHD by-pass system. Based on

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Nomenclature

B	magnetic field
E	electric field
J	current density
K	voltage drop across the load
l	channel length
m	mass flow rate
M_o	inlet Mach number
$\hat{P}$	total electric power
r	resistance/length
u	velocity
y	channel width
y_0	channel width at inlet

Greek symbols

δ	interaction parameter
γ	ratio of specific heats (=5/3 for all results in this work)
σ	electrical conductivity

these estimates it is possible to limit the range and number of design variables, thus ensuring the feasibility of using more advanced design/optimization tools for meeting the engineering constraints and requirements of the overall system. Refs. [1–3] investigated the problem of optimizing MHD generators under various constraints based on simplified assumptions to make the problem tractable. Analytical expressions were obtained for the optimum value of the desired cost function (total power extracted or optimum shape of the duct). While these analytical expressions provide useful insight into the influence of various fluid-dynamic and MHD parameters, they are of limited usefulness for design/analyses of practical MHD-based systems wherein the channel geometry (cross-section), electrical conductivity and possibly magnetic field might have a spatially variation. For instance, the simplified electrical conductivity models used in Refs. [1–3] are inadequate for MHD generators proposed for use in aerospace applications, where the cold incoming gas stream is ionized by non-equilibrium means [16]. Thus, to study MHD generators with an arbitrary electrical conductivity model or geometry, one has to take recourse to numerical methods. In this paper we develop a numerical methodology for coupling an optimization code with a flow solver, which enables it to address several problems of interest in MHD generator design and optimization.

Electrical conductivity of the flowing gas stream is perhaps the most important aspect in the design and optimization of MHD generators. Keeping this in mind, we demonstrate the use of our numerical scheme to study the problem of optimizing the power extracted from an ideal segmented Faraday generator with spatially varying electrical conductivity, by extending Neuringer's [1] analysis. We investigate the case of compressible inviscid plasma flow in a channel of fixed length and uniform cross-sectional area with a uniform transverse magnetic field, as discussed in [1]. We have chosen Neuringer's study to validate our numerical approach by comparing our results to analytical solutions derived by Neuringer. Further, we extend Neuringer's analyses for compressible flows to include the effects of variable electrical conductivity since it is relevant to flows in aerospace applications. We demonstrate the use of a compressible Euler flow solver coupled to an optimization code to accomplish this extension of Neuringer's analyses. The publicly available optimization code called ADS (Automatic Design Synthesis) [15] is used in our study. This paper is organized as follows. The formulation of the optimization problem is discussed in Section 2 along with the simplifying assumptions. Section 3 describes the numerical method and its generality to address more complicated analysis of MHD generators including the effects of space charge, Hall currents and friction/heat transfer effects.

Results and discussions are presented in Section 4. In this section, we validate our numerical approach by comparing our results with analytical solutions obtained by Neuringer [1]. Further, we present results for an ideal Faraday generator where a hypothetical electrical conductivity distribution is maintained by e-beams. Section 5 discusses the conclusions of this work.

2. Problem statement

Following Ref. [1], we present the simplified analysis of power extracted in a MHD generator. Fig. 1 shows a simplified MHD channel with constant cross-section. The magnetic field B is directed into the plane of the paper. Since the electrical conductivity and magnetic Reynolds number is small for practical MHD generators, the effect of the generated currents on the applied magnetic field are ignored and hence the applied magnetic field is assumed to be constant.

It is assumed that the flow is one-dimensional (no variations of the fluid-dynamical variables in the y and z directions). For an ideal segmented Faraday generator, induced currents in the

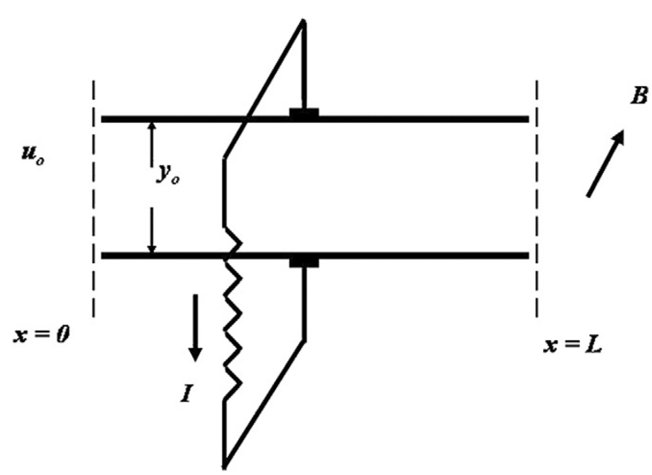


Fig. 1. Simplified diagram of an MHD generator (ideal segmented Faraday generator).

x-direction are small and hence Hall effects are ignored. The plasma is assumed to be electrically neutral; hence there are no space-charge sheaths near the walls. Friction and other heat losses are also ignored. Based on these assumptions it is possible to use Kirchoff's voltage law to express a relationship between the induced electric field (E) and the current density (J). If $r(x)$ represents the external unit resistance at a location 'x' along the channel, $J(x)$ represents the induced current per unit length of channel at location 'x', σ represents the plasma conductivity in the channel and y represents the channel width

$$Ey = Jr + \frac{Jy}{\sigma} \tag{1}$$

where $E = uB$ is the induced electric field, Jr is the voltage drop in the external resistance and Jy/σ is the voltage drop due to the internal resistance of the plasma. If the drop in voltage across the external resistance is denoted by k, then the above equation can be rewritten in terms of the current density as follows

$$J = \frac{\sigma}{y}\{uBy - k\} \tag{2}$$

The power generated per unit length of the generator would then be

$$\frac{\hat{P}}{\text{length}} = rJ^2 = kJ = k\sigma\left\{uB - \frac{k}{y}\right\} \tag{3}$$

The total power generated in a channel with length l would then be

$$\hat{P} = k\int_0^l \sigma\left(uB - \frac{k}{y}\right)\mathrm{d}x \tag{4}$$

The optimization problem seeks to determine the axial velocity distribution $u(x)$ and the voltage drop in the external resistance k, in order to maximize $\hat{P}$.

Neuringer [1] used the simplified model for current flow described above to obtain the optimum power extracted from an inviscid compressible flow in a MHD channel with constant cross-section and electrical conductivity, subjected to a uniform transverse magnetic field. Neuringer showed that the power delivered to the external load equals the difference between the total enthalpy flux between the inlet and exit of the channel. Hence, for a given inlet enthalpy flux, the power delivered to the external load is at a maximum, if the enthalpy flux at the channel exit would be minimum. Using this argument, Neuringer derived analytical expressions for optimum power generated in a MHD generator, for a prescribed set of conditions (γ, M_o, y_o and δ). Neuringer also derived analytical expressions for axial variation of velocity, Mach number and pressure, corresponding to maximum power extraction. Details of this derivation are explained in Ref. [1].

3. Numerical method

In this section, we discuss a numerical approach to study the problem of optimum power extraction wherein the cross-sectional area, electrical conductivity and magnetic field may be spatially varying (derivation of power generation shown in Eq. (4) does not require the cross-sectional area, magnetic field or electrical conductivity to be constant). We demonstrate the use of this methodology to study the above-mentioned optimization problem. In our numerical approach, a compressible Euler flow solver is coupled to an optimization code, to evaluate the optimum value of extracted power given in Eq. (4). The terminal voltage of the generator (k), is the design variable and power generation the objective function in the optimization routine. For a given initial flow field, terminal voltage (k), spatial distribution of electrical conductivity, and/or magnetic field, a spatially varying current is computed using the expression (2). Knowing the spatial (pointwise) distribution of current, the Lorentz force ($J.B$) in the momentum equation is evaluated. Similarly, work done by Lorentz forces and the Joulean dissipation ($uJB + \frac{j^2}{\sigma}$) needed in the energy equation is also evaluated using Eq. (2). Having computed the MHD related source terms in the momentum and energy equations, an inviscid compressible flow solver is used to obtain converged solution of the velocity field. Based on the velocity distribution computed using the flow solver, the total power generation is calculated using Eq. (4). The total power and design variable k, are then used as inputs to an optimization routine. The optimization routine generates subsequent values of k, and computes the corresponding values of power generated. The optimization routine converges when the optimum power corresponding to a given set of prescribed conditions (inlet Mach number, B-field, channel geometry and electrical conductivity distribution) is reached. Fig. 2 shows the flow chart of the numerical procedure used in this work.

It must be pointed out that the above-mentioned numerical framework allows a more realistic model to evaluate the electric current (including the effect of Hall current), which can then be used to obtain the total power as $P = \int_V J.E\,\mathrm{d}V$. This can be accomplished by use of a Poisson solver to compute the pointwise current and electric field distribution and hence the total

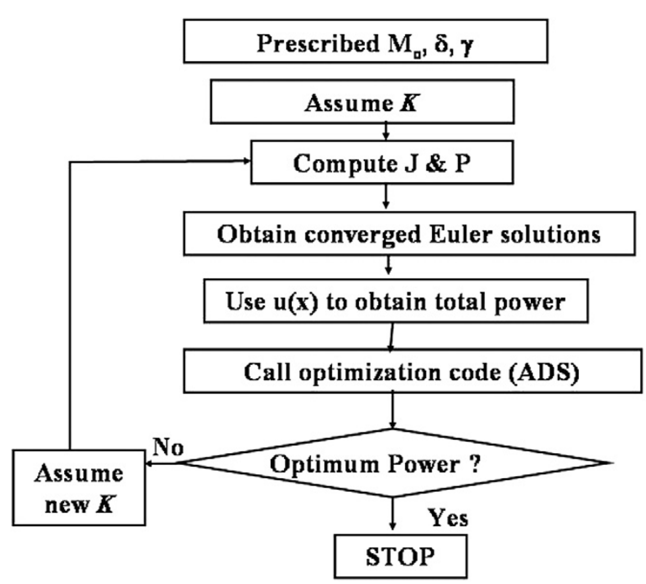


Fig. 2. Outline of the numerical Euler solver/optimizer strategy.

power extracted in the MHD generator. The electric field may be derived from a scalar potential, namely, $\vec{E} = -\vec{\nabla}\varphi$. Ohm's law may then be used to relate this potential to the current density as shown below.

$$\vec{J} = \sigma(-\vec{\nabla}\varphi + \vec{V} \times \vec{B}) - \frac{\omega\tau}{B}(\vec{J} \times \vec{B}) \tag{5}$$

where σ denotes the electrical conductivity, $\omega\tau/B$ is the Hall parameter, J is the electric current density and B is the applied magnetic field vector. Since the induced magnetic field is neglected, the only electromagnetic field quantity that needs to be computed numerically is the electric potential. When the divergence of the above expression for current is set to zero and is re-written in terms of the electric potential, a Poisson type equation is obtained. Solution of this Poisson equation yields the electric potential φ and electric current density J. The Joule heating term and Lorentz forces in the energy and momentum equations can thus be obtained knowing the current distribution. Effects of space charge can also be included in the above formulation. Effects of viscosity and heat transfer could also be included if one were to use a Navier–Stokes flow solver to compute the flow field. The objective function (Eq. (4)) would be modified to reflect these losses. The power loss due to friction and heat transfer would be subtracted from Eq. (4) to yield a modified objective function, which would then be optimized. Use of more complex flow and current models increase the computational burden and hence should be used in the later stages of design/optimization, if need be.

In this paper, however, we use the simple model proposed by Neuringer (as described in Eqs. (1)–(4)) to demonstrate the use of our numerical approach to study a MHD generator with spatially varying electrical conductivity. The channel cross-section and B-field are assumed to be constant. The magnetic field was assumed to be 1 T. All the flow computations in this work were performed using a 1-D Euler code for a channel length of 1 meter. A hundred grid points were used for the flow computations. Grid convergence studies showed that a hundred grid points were adequate for resolving the flow gradients. A typical flow/optimization solution took about 2–3 minutes on a single CPU machine. The Euler solver was converged to 8 orders of magnitude for each value of the design variable k. The ADS code is used to perform the optimization. The ADS code solves the non-linear constrained optimization problem

$$\text{Minimize } f(\vec{X}) \tag{6}$$

subject to the inequality constraints

$$G_j(\vec{X}) \leqslant 0, \quad j = 1, N_j$$

and equality constraints

$$H_k(\vec{X}) = 0, \quad k = 1, N_k$$

where the vector $\vec{X} = (x_1, x_2, x_3, \ldots, x_N)$ is the vector of design parameters. ADS employs the method of steepest descent which leads to the iterative solution procedure for $\vec{X}$ given by

$$\vec{X}^{n+1} = \vec{X}^n + c.\vec{S}^n$$

where the superscript n is the iteration counter (optimization cycle), $\vec{S}$ is the vector search direction, and c is a scaling parameter. The optimization routine converges when the relative and absolute change in the objective function are less than a user-defined convergence criterion. If $F(0)$ were to be the value of the objective function corresponding to the initial design, the absolute change in the objective function is defined as $\varepsilon F(0)$. In this work, ε was set equal to 0.001. The relative change in the objective function is defined as $\|(f(\vec{X}^{n+1}) - f(\vec{X}^n))/(f(\vec{X}^n))\|$. The convergence criterion for relative change in the objective function was set equal to 0.001.

4. Results and discussion

This section discusses the following:

(a) Validation of our numerical approach.
(b) Use of the coupled Euler-ADS numerical scheme to study MHD power generation with spatially varying electrical conductivity.

4.1. Validation of our numerical approach

In order to validate our numerical approach we compared the non-dimensional voltage drop across the load (k') and optimum power extracted with analytical expressions proposed by Neuringer for a MHD channel with constant area and constant electrical conductivity. Analytical results were obtained by a solution of Eq. (19) and Eq. (23) in Ref. [1] using an iterative scheme. This validation was done for a range of supersonic inlet Mach numbers and interaction parameters as shown in Table 1. The interaction parameter is defined as the ratio between the electrical body force to the inertial force and is expressed as [$\delta = \frac{B^2\sigma l}{\rho u}$].

Fig. 3 shows a comparison between the axial variation of Mach number for optimum power extraction for $M_o = 4.5$, $\delta = 3.0$ using analytical results from [1] and our coupled

Table 1
Comparison of numerical optimization results with the analytical results

M	δ	k' (analytical)	k' (numerical)	Optimum power in MW (analytical)	Optimum power in MW (numerical)
3.0	0.5	1.851	1.852	1.355	1.350
3.0	1.0	1.686	1.686	1.917	1.910
3.0	2.0	1.548	1.551	2.247	2.237
3.0	3.0	1.513	1.512	2.309	2.294
3.0	4.0	1.505	1.504	2.321	2.315
3.0	8.0	1.502	1.502	2.324	2.323
4.5	0.5	1.882	1.891	4.702	4.735
4.5	1.0	1.755	1.757	6.913	6.933
4.5	2.0	1.621	1.622	8.462	8.484
4.5	3.0	1.578	1.579	8.841	8.847
4.5	4.0	1.560	1.567	8.936	8.946
4.5	8.0	1.556	1.554	8.97	8.966
6.0	0.5	1.890	1.898	11.307	11.278
6.0	1.0	1.776	1.774	16.777	16.785
6.0	2.0	1.6468	1.651	20.793	20.822
6.0	3.0	1.6012	1.602	21.844	21.858
6.0	4.0	1.5844	1.590	22.129	22.130
6.0	8.0	1.5751	1.575	22.238	22.232

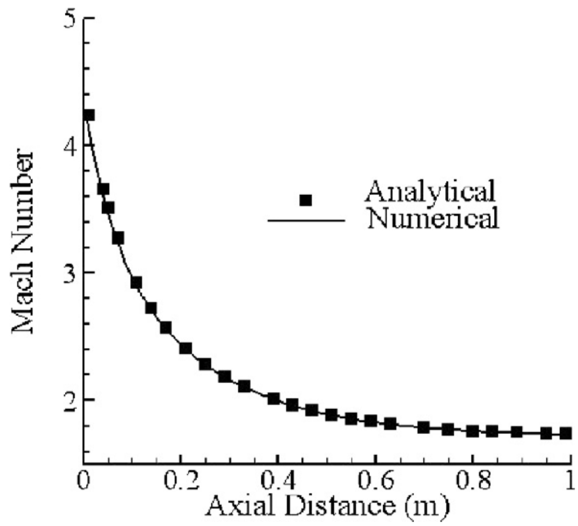
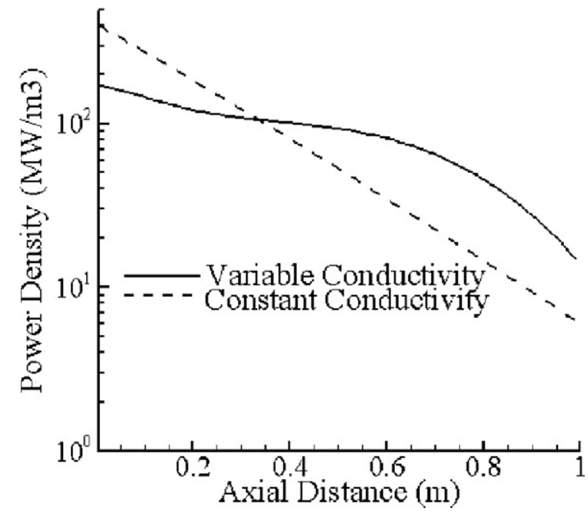


Fig. 3. Comparison of Mach number for $M_o = 4.5$, $\delta = 3$ using analytical and numerical methods.

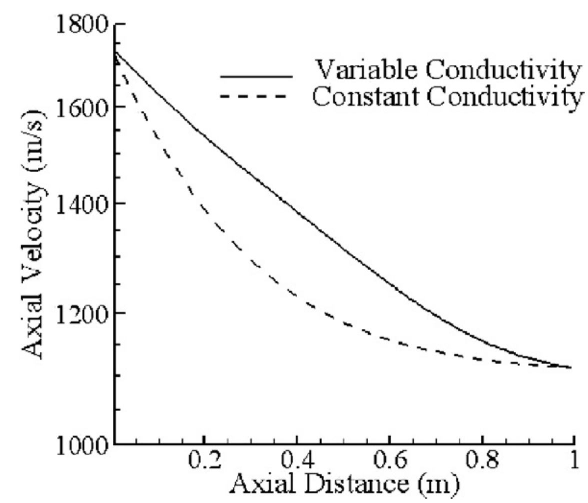
Euler/optimizer numerical approach. From Table 1 and Fig. 3 excellent agreement is seen between the results obtained from our numerical approach and analytical predictions for a range of Mach numbers and interaction parameters, thus validating our numerical approach and its implementation. Fig. 3 shows that the Mach number drops sharply near the inlet and reaches an asymptotic value. This characteristic is opposite to subsonic flows wherein the Mach number (and velocity) increases sharply near the exit (as shown in Fig. 4 Ref. [1]). This behavior has important implications with regards to the uniformity of power distribution along the channel, as discussed next.

4.2. Optimization of a constant area channel with varying electrical conductivity

From Eq. (4) is it seen that for an MHD channel with constant electrical conductivity, magnetic field and cross-section, the power density varies axially as the velocity—hence the power delivered to the external load takes place in the regions with the largest velocity gradients. Thus, for maximum power extraction at a given δ, a major portion of the power delivered to the external load takes place near the channel extremities (inlet for supersonic flows as shown in Fig. 3) and outlets (for subsonic flows). This is a major drawback in MHD generators with constant electrical conductivity. This situation can be remedied if the electrical conductivity of the working fluid (flowing plasma) is also spatially varying. A more uniform power density distribution could be obtained by using a lower value of electrical conductivity in regions where the velocity gradient is steep and vice versa. Hence for MHD generators with supersonic inlets, a lower value of electrical conductivity near the inlet could compensate for the steep velocity gradient, thus leading to a more uniform power-density distribution along the MHD channel. In this section we investigate the effect of spatially varying electrical conductivity on the power density distribution along the channel, as compared to a case with constant conductivity; using our coupled Euler–optimizer numerical approach. The total optimum extracted power is kept the same in both cases. Our numerical approach is general enough to study any arbitrary model for electrical conductivity and is not limited to the hypothetical electrical conductivity distribution described next.



(a)



(b)

Fig. 4. Axial variation of power density (a) and velocity (b) for $P_{opt} = 8.8$ MW, $M_o = 4.5$ with constant σ and axially varying σ.

In aerospace applications, where it is proposed that electrical conductivity be sustained by non-equilibrium ionization techniques, it is possible, in theory, to obtain an axially varying electrical conductivity distribution. For MHD by-pass engines it has been shown that e-beams [16] are perhaps the best non-equilibrium ionizing method. The electron number density and hence electrical conductivity generated by using e-beams is proportional to the current of the e-beam and the density along the channel [17]. E-beams can thus, in theory, create a region of spatially varying electrical conductivity. Axial variation of electrical conductivity along the length of the channel can be generated by varying the e-beam current. The e-beam current can be kept constant for a prescribed length (x_1) to obtain a constant electrical conductivity and then increased axially to obtain a spatial distribution of electrical conductivity of the form shown below.

$$\sigma = \sigma_1 \quad \text{for } 0 < x < x_1 \tag{7a}$$

$$\sigma = \sigma_1 + \tau(x - x_1)^2 \quad \text{for } x_1 < x < l \tag{7b}$$

The above form of electrical conductivity distribution is chosen so as to maintain a lower value of electrical conductivity near the inlet where there are steep velocity gradients and rapidly increase it when the velocity gradients in the channel reach an asymptotic value (see Fig. 4(b)). The values of σ_1, τ and x_1 can be varied based on system design parameters and engineering constraints. In this paper we choose values of σ_1, τ and x_1 to match the optimum power extracted from a channel of constant conductivity. As an illustrative example, we present a case where the inlet Mach number (M_o) is 4.5 and the optimum power extracted is 8.8 MW. For the case with constant electrical conductivity, the interaction parameter was 3, corresponding to a constant electrical conductivity of 522 mho/m (it is not possible to define a global interaction parameter when the electrical conductivity is varying across the channel). For the case with varying electrical conductivity the following values were used to generate a spatially varying electrical conductivity distribution discussed above; $\sigma_1 = 250$, $x_1 = 0.15$, and $\tau = 1200$. The value of x_1 was chosen based on the approximate length over which the velocity drops sharply. As seen in Fig. 4(b), there is a steep velocity gradient for $x \cong$ less than 0.15, hence x_1 was set equal to 0.15. The values of σ_1 and τ were based on keeping the overall conductance (in mho) of the channel $\int_0^i \sigma \, dx$, the same for both the cases (so as to keep the total extracted power the same).

Fig. 4(a) and (b) show the comparison of axial power density (W/m^3) and velocity for constant and axially varying σ, respectively. It is seen that when the electrical conductivity along the channel is constant, the power density is highly non-uniform, with most of the power extraction taking place within half the channel length near the inlet. The power density near the inlet is almost two orders of magnitude higher than that near the exit plane. Using an axially varying electrical conductivity, it is possible to achieve a fairly constant velocity gradient (Fig. 4(b)), leading to a greater uniformity in the power density distribution along the channel. The variation in power density can almost be reduced by one order of magnitude using variable electrical conductivity in the channel. Using the numerical approach outlined in this work, different conductivity models and/or channel geometries can be investigated. Since the computational effort is small, the designer can investigate several scenarios before attempting to incorporate other effects such as friction, heat transfer, Hall effects and space-charge.

5. Conclusions

In this paper, optimum power extraction in an MHD generator with supersonic inlet Mach numbers was examined when the electrical conductivity of the working fluid is spatially varying. A numerical approach was developed wherein a flow solver was coupled to an optimization routine to compute the optimum power extraction and the variation of fluid-dynamical variables corresponding to this optimum value. The numerical approach is general enough to include any arbitrary conductivity model, and also the effects such as heat transfer, friction, space-charge and Hall currents. The numerical approach was applied to a simplified ideal Faraday generator discussed by Neuringer. It was shown that MHD generators with axially varying electrical conductivities (varying interaction parameters) can have a more uniform power density distribution along the length of the channel, as compared to MHD generators with constant electrical conductivities.

References

[1] J.L. Neuringer, Optimum power generation from a moving plasma, J. Fluid Mech. 7 (1960) 287–301.

[2] D.T. Swift-Hook, J.K. Wright, The constant Mach-number MHD generator, J. Fluid Mech. 15 (1962) 97–110.

[3] C. Carter, The optimization of a magnetohydrodynamic generating duct, Brit. J. Appl. Phys. 17 (1966) 863–871.

[4] I. Inoue, M. Ishikawa, J. Umoto, Numerical study of arc phenomena in boundary layer on MHD generator, Energy Conversion and Management 33 (9) (1992) 873–884.

[5] Y. Inui, H. Ito, T. Ishida, Two dimensional simulation of closed cycle disk MHD generator considering nozzle and diffuser, Energy Conversion and Management 45 (13–14) (2004) 1993–2004.

[6] M. Ishikawa, Y. Koshiba, T. Matsushita, Effects of induced magnetic field on large scale pulsed MHD generator with two phase flow, Energy Conversion and Management 45 (5) (2004) 707–724.

[7] B.K. Sawhney, S.S. Verma, Optimum power generation characteristics of an MHD generator with kerosene oil-heavy fuel oil mixture as working fluid, Energy Conversion and Management 36 (5) (1995) 343–353.

[8] B. Sahin, A. Kodal, H. Yavuz, A performance analysis for MHD power cycles operating at maximum power density, J. Phys. D: Appl. Phys. 29 (1996) 1473–1475.

[9] B. Sahin, A. Kodal, A.S. Oktem, Optimal performance analysis of irreversible regenerative MHD power cycles, J. Phys. D: Appl. Phys. 32 (1999) 1832–1841.

[10] L. Chen, J. Gong, F. Sun, C. Wu, Heat transfer effect on the performance of MHD power plant, Energy Conversion and Management 43 (15) (2002) 2085–2095.

[11] G. Ibáñez, S. Cuevas, M.L. de Haro, Optimization analysis of an alternate magnetohydrodynamic generator, Energy Conversion and Management 43 (14) (2002) 1757–1771.

[12] V.S. Bajovi, A reliable tool for the design of shape and size of Faraday segmented MHD generator channel, Energy Conversion and Management 37 (12) (1996) 1753–1764.

[13] V.L. Fraishtadt, A.L. Kuranov, E.G. Sheikin, Use of MHD systems in hypersonic aircraft, Technical Physics 43 (11) (1998) 1309–1313.

[14] E.P. Gurijanov, P.T. Harsha, AJAX: New directions in hypersonic technology, AIAA Paper 96-4609.

[15] G.N. Vanderplaats, ADS—A Fortran program for automated design synthesis—Version 1.10, NASA CR-177985, 1985.

[16] S.O. Macheret, M.N. Shneider, R.J. Miles, Potential performance of supersonic MHD power generators, AIAA-2001-0795, in: AIAA 39th Aerospace Sciences Meeting & Exhibit, Reno, NV, 2001.

[17] A.L. Kuranov, E.G. Sheikin, MHD control on hypersonic aircraft under "AJAX" concept. Possibilities of MHD generator, in: 40th AIAA Aerospace Sciences Meeting, Reno, NV, 2002.